THEORY OF DARK ENERGY BY GRAVITATIONAL COMBUSTION AND QUANTUM SELF-REPLICATION: A SCALAR-TENSORIAL ALTERNATIVE FRAMEWORK

Unifying black hole radiation, the accelerated
expansion of the Universe, and the microphysics of dark energy

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Abstract. An alternative, self-consistent theoretical framework is presented that identifies dark energy with a real scalar field ϕ generated as a by-product of the *gravitational combustion* of matter in high-curvature regions, predominantly at the event horizons of black holes. The field ϕ is endowed with a non-conventional self-interaction potential $V_{\rm rep}(\phi) = (\lambda_{\rm rep}/4!)\,(\phi^2 - \phi_c^2)^2\,\Theta(\phi - \phi_c)$ that activates above a critical threshold ϕ_c, inducing a local tachyonic instability. This instability translates into a *quantum self-replication* mechanism in which the collision of two quasiparticles of the ϕ field produces additional pairs at a rate $\Gamma_{\rm rep} = |V''_{\rm eff}(\phi)|/(4\pi)$. Starting from a scalar-tensorial Lagrangian $\mathcal{L} = \mathcal{L}_{\rm EH} + \mathcal{L}_\phi + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm BH} + \mathcal{L}_{\rm rep}$, the coupled system of modified Einstein–Klein–Gordon equations is rigorously derived, and applied to FLRW cosmology, yielding modified Friedmann equations with a source term. It is shown that, in the dark-energy-dominated regime, self-replication produces an exponential growth $\rho_\Lambda(t) \propto \mathrm{e}^{(\Gamma_{\rm rep} - 3H)\,t}$ that drives a super-accelerated expansion $a(t) \propto \exp[\exp(\Gamma_{\rm rep}\,t/2)]$ *without* violating the weak energy condition. The complete microphysics (quasiparticle spectrum, cross section, mean free time, coherence length) is developed, and observational predictions for $H(z)$, $w(z)$, and gravitational-wave signatures are obtained. The theory is falsifiable: any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in the range $0 < z < 2$ refutes the model.

Keywords: dark energy; black holes; gravitational combustion; quantum self-replication; scalar field; FLRW cosmology; modified Friedmann; tachyonic instability; equation of state; gravitational waves.

1 Introduction

The nature of dark energy constitutes, to date, the fundamental unsolved problem of theoretical cosmology. Since the pioneering observations of Riess *et al.* [1] and Perlmutter *et al.* [2], which independently demonstrated the late-time cosmic acceleration through type Ia supernova light curves, the standard cosmological model ΛCDM has consolidated a remarkable body of observational evidence: anisotropies of the cosmic microwave background measured by Planck [3], baryon acoustic oscillations (BAO) in the galaxy distribution [4], and the large-scale structure of the Universe [5]. However, despite its phenomenological success, the ΛCDM model suffers

from two deep conceptual problems that the present theory aims to address: the *cosmological constant problem* and the *cosmic coincidence problem*.

The cosmological constant problem, classically formulated by Weinberg [6], consists of the discrepancy of approximately 120 orders of magnitude between the value of the vacuum energy density predicted by quantum field theory (estimated as $\rho_{\rm vac} \sim M_{\rm Pl}^4$ in natural units) and the observed value $\rho_\Lambda^{\rm obs} \sim (2.3 \times 10^{-3}\,{\rm eV})^4$. This discrepancy, far from being a technical detail, calls into question the consistency between quantum mechanics and general relativity in the ultraviolet energy regime. Proposed solutions — draconian renormalization, anthropic principles, exact supersymmetry — have failed to produce testable predictions.

The cosmic coincidence problem, in turn, questions why the dark energy density ρ_Λ and the matter density $\rho_{\rm m}$ are of the same order of magnitude precisely at the present epoch, given that they scale very differently with the scale factor $a(t)$. If dark energy were a true vacuum constant, this balance would occur only in an extraordinarily narrow cosmic window, which suggests — in the view of a growing fraction of the cosmological community — that ρ_Λ is not static but evolves dynamically.

It is in this context that the present theory is framed. We start from a phenomenological premise suggested by elementary physical analogies: *any matter undergoing an irreversible process of energy conversion emits a low-energy residual by-product*. Just as a match, when lit, emits light and heat but also smoke — low-temperature residual particles that disperse into the environment — so too any matter falling into an extreme gravitational field, in particular matter crossing the event horizon of a black hole, should emit a residual radiation associated with the very process of capture. This residual radiation, we contend, is precisely dark energy.

The hypothesis is not entirely new in its inspiration: Hawking [7, 8] showed that black holes emit thermal radiation with temperature $T_{\rm H} = \hbar c^3/(8\pi G M k_B)$ due to quantum effects near the horizon. However, Hawking radiation is thermal with a black-body spectrum that decreases rapidly for astrophysical black holes (with $T_{\rm H} \sim 10^{-8}\,{\rm K}$ for a solar-mass black hole), and its total energy density integrated over the history of the Universe is numerically insufficient to explain $\rho_\Lambda^{\rm obs}$. The present theory differs in two essential points: (i) it postulates that the emission is not strictly thermal but includes a *coherent scalar channel* dominated by a field ϕ whose characteristic wavelength is of the order of the current cosmic radius; and (ii) it introduces a *quantum self-replication* mechanism by which the density of the ϕ field grows exponentially once a critical threshold ϕ_c is exceeded.

The *balloon* analogy deserves a mathematical clarification developed in Section 2: the Universe is modeled as a four-dimensional closed manifold of topology $\mathbb{R} \times S^3$ (spatially compact), with FLRW metric and curvature $k = +1$. The *smoke* filling the balloon is the ϕ field; the *internal pressure* dilating the balloon is the negative pressure $p_\phi = -\rho_\phi$ in the dominant self-replication regime. The question *why does the Universe inflate ever faster?* finds an answer in the modified continuity equation

$$\dot\rho_\phi + 3H(1 + w_\phi)\rho_\phi = \Gamma_{\rm rep}\rho_\phi, \tag{1}$$

where $\Gamma_{\rm rep}$ is the self-replication rate. When $\Gamma_{\rm rep} > 3H(1 + w_\phi)$, the right-hand side dominates and ρ_ϕ grows exponentially; through the first Friedmann equation $H^2 \propto \rho_\phi$, this implies that the Hubble rate H itself grows, producing a super-accelerated expansion *without* requiring a *phantom* equation of state $w < -1$ that would violate the weak energy condition.

The structure of the article is as follows. Section 2 states the four fundamental postulates and defines the terminology. Section 3 constructs the complete Lagrangian from first principles. Section 4 derives the equations of motion via the action principle. Section 5 presents the conceptual core of the work: the quantum self-replication mechanism. Section 6 applies the model to FLRW cosmology. Section 7 rigorously demonstrates the super-exponential acceleration. Section 8 develops the detailed microphysics. Section 9 presents the observational predictions.

Section 10 offers a critical analysis. Section 11 synthesizes the conclusions. Appendices A and B collect notation and auxiliary derivations.

2 Conceptual foundations of the theory

2.1 Fundamental postulates

The theory developed here rests on four physical postulates stated below. Each postulate is formulated as a precise mathematical statement, susceptible to translation into a Lagrangian expression in Section 3. It should be noted from the outset that the postulates are *physical hypotheses* proposed by the theory; their definitive validation depends on the observational confrontation discussed in Section 9.

Postulate P1 (Gravitational combustion)

Any portion of matter subjected to a gravitational acceleration $|\mathbf{g}|$ exceeding a critical threshold $g_c \sim cH_0$ emits a real low-energy scalar field ϕ, called *dark radiation*, as the inevitable and irreversible by-product of the energy conversion process associated with the local non-stationary metric. The emission rate per unit volume is proportional to the local matter density and to the scalar curvature R:

$$\mathcal{Q}_\phi(\mathbf{x}, t) = \xi_{\rm BH}\, \rho_{\rm m}(\mathbf{x}, t)\, |R(\mathbf{x}, t)|^{1/2},$$

where $\xi_{\rm BH}$ is a dimensional coupling fixed in Section 8 by normalizing to the observed density $\rho_\Lambda^{\rm obs}$.

The phenomenological justification of Postulate P1 is threefold. First, Hawking radiation [7] already establishes that event horizons emit; the present postulate generalizes this result to any region where the local curvature is sufficiently high. Second, the thermalization of matter falling into a black hole — a process well modeled by the Shakura–Sunyaev accretion disks [9] — implies an irreversible energy conversion that, by the second law of thermodynamics, must produce entropy; the field ϕ can be interpreted as the *entropic carrier* of that conversion. Third, the analogy with ordinary thermodynamic processes (chemical combustion produces heat, light, and smoke) suggests that a low-energy by-product is generic in any irreversible conversion process.

Postulate P2 (Dominance of black holes)

Among all sources of gravitational combustion, black holes — stellar, intermediate, and supermassive — are the dominant contributions to the total dark energy budget of the Universe. The physical reason is that the product $\rho_{\rm m}\,|R|^{1/2}$ is maximized at event horizons, where $\rho_{\rm m}$ is large (accreting matter) and $|R| \sim 1/r_s^2$ with $r_s = 2GM/c^2$.

Postulate P2 is quantitative: the fraction of dark energy produced by sources not associated with black holes (neutron stars, white dwarfs, dense interstellar gas regions) is of order 10^{-4} relative to the black-hole contribution, as shown in Appendix B (Section B.1). This justifies referring to the main source hereafter as $\rho_{\rm BH}$ and allows the source balance equation (Section 4) to be written without lower-order additional terms.

Postulate P3 (Self-interacting scalar field)

Dark radiation is described by a real scalar field $\phi : \mathcal{M} \to \mathbb{R}$ over the spacetime manifold $\mathcal{M}$, endowed with an effective potential $V_{\rm eff}(\phi)$ containing: (i) a mass term m_ϕ, (ii) a quartic self-interaction $\lambda\,\phi^4/4!$, (iii) a non-minimal coupling to curvature $\xi R\phi^2/2$, and (iv) a self-replication term $V_{\rm rep}(\phi)$ that activates above a critical threshold ϕ_c.

Postulate P3 fixes the Lagrangian structure: the field ϕ is the fundamental dynamical degree of freedom, and all cosmological phenomena of the theory follow from its evolution. The introduction of a non-minimal coupling $\xi R\phi^2$ follows the tradition initiated by Chernikov–Tagirov [10] and developed by Ford, Starobinsky, and others in the context of inflation [11]. The novelty with respect to conventional quintessence models [12] lies in the term $V_{\rm rep}(\phi)$, whose precise form is discussed in Postulate P4.

Postulate P4 (Collision-induced self-replication)

When the local amplitude of the ϕ field exceeds the threshold ϕ_c, the effective potential $V_{\rm eff}(\phi)$ develops a localized tachyonic region, inducing a Bose–Einstein instability that manifests as pair production via a gravitational analogue of the Schwinger effect. The production rate per unit volume is proportional to the local number density n_ϕ and to the collision cross section $\sigma_{\rm rep}$, yielding a balance equation $\dot{n}_\phi = \Gamma_{\rm rep}\, n_\phi$ with $\Gamma_{\rm rep} > 0$.

Postulate P4 is the central innovation of the present theory. Its detailed microscopic justification is developed in Section 5; here it suffices to note that it rests on three ingredients: (a) quantum field theory in curved spacetime, in particular pair production by external fields [25, 26]; (b) the Higgs tachyonic instability analogous to that of the Standard Model in strong background fields [29]; and (c) the Schwinger effect generalized to gravitational fields [27, 28]. The combination of these three ingredients within a cosmological framework is, to the best of our knowledge, original to this theory.

2.2 Terminology

To fix the language, the following technical terms are introduced and used throughout the work:

- **Gravitational combustion:** any irreversible physical process in which matter at rest is partially converted into energy of the ϕ field, analogous to the chemical combustion process that produces heat and by-products.
- **Dark radiation:** the scalar field ϕ itself, in its capacity as a by-product of gravitational combustion. It should not be confused with *dark matter*, which is a distinct component of presumably particle nature.
- **Quantum self-replication:** a mechanism by which two quasiparticles of the ϕ field, upon colliding when the local density exceeds the threshold ϕ_c, produce additional pairs of quasiparticles of the same field, at a rate $\Gamma_{\rm rep} > 0$.
- **Holographic pressure:** the negative effective pressure $p_\phi = -\rho_\phi$ exerted by the ϕ field on the geometry of the Universe, so called because it is proportional to the information stored on the surface of the apparent horizon, in line with the holographic principle [23, 24].
- **Critical threshold ϕ_c:** the value of the field amplitude above which the term $V_{\rm rep}(\phi)$ activates; it is computed explicitly in Section 5 from the model parameters.

2.3 Physical analogies and their limits

The *match–smoke* analogy motivating Postulate P1 is conceptually powerful but must be used with caution. In chemical combustion, smoke consists of macroscopic aerosols of defined composition; in gravitational combustion, the *smoke* is a coherent quantum field. The essential difference lies in the fact that chemical smoke is an *inert* by-product that only disperses and dilutes, whereas the field ϕ, by its nature as a self-interacting quantum field, can *self-replicate* as postulated in P4. This difference is what endows the theory with its explanatory power regarding cosmic acceleration: unlike chemical smoke, whose density only decreases with the expansion of

the containing volume, the density of the ϕ field can grow in absolute terms, thereby generating an increasing internal pressure that inflates the universal *balloon* at an accelerating rate.

Similarly, the image of the Universe as a *balloon* deserves a mathematical clarification. The Universe is modeled with the closed FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-r^2} + r^2\,\mathrm{d}\Omega^2\right], \tag{2}$$

with spatial curvature $k = +1$. The total comoving volume is $V_{\mathrm{comoving}} = 2\pi^2$ (in units of the curvature radius); the corresponding physical volume is $V_{\mathrm{physical}}(t) = 2\pi^2 a^3(t)$. The image of the *balloon filling with smoke* translates into the fact that the physical density of the ϕ field, $\rho_\phi(t)$, does not simply decrease as a^{-3} (as non-relativistic dust would) or as a^{-4} (as radiation does), but rather, in the dominant self-replication regime, obeys the exponential law derived in Section 7.

2.4 Comparison with preceding theoretical frameworks

The present theory bears partial relationships with several contemporary research lines, from which it differs by essential features:

Quintessence. Quintessence models [12, 13] describe dark energy as a scalar field with a potential $V(\phi)$ that evolves slowly. They differ from the present theory in that (i) they do not postulate a production mechanism for ϕ by gravitational combustion, and (ii) they do not include a self-replication term $V_{\mathrm{rep}}(\phi)$. In conventional quintessence ρ_ϕ decreases with $a(t)$, whereas here ρ_ϕ can grow exponentially.

k-essence. These models [16] introduce non-canonical kinetic terms $K(\phi, X)$ with $X = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$. The present theory uses canonical kinetics $-\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$ and places the novelty in the potential $V_{\mathrm{rep}}(\phi)$; the difference is conceptual: k-essence modifies the kinetics, the present theory modifies the production mechanism.

$f(R)$ theories. Modifications to Einstein gravity via $f(R)$ [17, 18] reformulate the cosmological constant as a modification of the gravitational action. The present theory keeps the standard Einstein–Hilbert action and attributes dark energy to a material field ϕ with novel dynamics. The two approaches are complementary and, in principle, combinable.

Holographic dark energy. Holographic models [19, 20] relate ρ_Λ to the size of the apparent horizon via $\rho_\Lambda \sim M_{\mathrm{Pl}}^2/L^2$. Although the present theory invokes the holographic principle in defining *holographic pressure*, it does not identify ρ_Λ with a horizon energy density but with a material field ϕ that accumulates cosmically.

Emergent gravity. Verlinde's proposal [21] and Padmanabhan's work [22] interpret gravity as an entropic force. The present theory does not adopt this ontological perspective but shares the intuition that thermodynamics underlies dark energy; in our case, the thermodynamics of the *combustion* of matter.

With this conceptual delimitation, we are ready to construct the fundamental Lagrangian of the theory.

3 Construction of the fundamental Lagrangian

3.1 Action principle and symmetries

The theory is defined by the action functional

$$\mathcal{S}[\,g_{\mu\nu}, \phi\,] = \int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu\phi, R) + \mathcal{S}_{\mathrm{mat}}\,[\,g_{\mu\nu}, \psi_{\mathrm{mat}}\,], \tag{3}$$

where $g = \det(g_{\mu\nu})$, R is the Ricci scalar of the Levi-Civita connection of $g_{\mu\nu}$, and $\mathcal{S}_{\rm mat}$ is the action of standard matter (including black-hole models). The *symmetries* required of the system are:

(S1) **General diffeomorphism invariance:** $\mathcal{S} \mapsto \mathcal{S}$ under $x^\mu \mapsto x'^\mu(x)$. This symmetry, characteristic of any relativistic theory of gravitation, ensures conservation of the energy-momentum tensor $\nabla^\mu T_{\mu\nu} = 0$ via Noether's identity.

(S2) **Invariance under $\phi \mapsto -\phi$:** the Lagrangian is even in ϕ, which rules out terms $\propto \phi^3$ or $\propto \phi$ *except* for the explicitly introduced source and self-replication terms. This parity is physically justified because the field ϕ describes a by-product with vanishing quantum number.

(S3) **Conformal reparametrization invariance (conditional):** the coupling $\xi = 1/6$ (conformal coupling) leaves the theory invariant under $g_{\mu\nu} \mapsto \Omega^2(x)\, g_{\mu\nu}$, $\phi \mapsto \Omega^{-1}(x)\,\phi$. This symmetry is broken by $V_{\rm rep}(\phi)$, which fixes the scale ϕ_c physically.

(S4) **Conservation of lepton and baryon number:** the field ϕ is neutral under all gauge groups of the Standard Model, implemented by requiring $\mathcal{L}_\phi$ to contain no derivatives of fermion fields.

3.2 Lagrangian decomposition

The total Lagrangian decomposes into five functionally independent contributions:

$$\mathcal{L} = \mathcal{L}_{\rm EH} + \mathcal{L}_\phi + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm BH} + \mathcal{L}_{\rm rep}. \tag{4}$$

Each term is constructed below.

3.2.1 Einstein–Hilbert term $\mathcal{L}_{\rm EH}$

The purely gravitational part is the standard Einstein–Hilbert action:

$$\mathcal{L}_{\rm EH} = \frac{M_{\rm Pl}^2}{2}\, R, \qquad M_{\rm Pl} = \frac{1}{\sqrt{8\pi G}}. \tag{5}$$

Here $M_{\rm Pl} \approx 2.435 \times 10^{18}\,\text{GeV}$ is the reduced Planck mass (in units $\hbar = c = 1$). No $f(R)$ modifications are introduced at this stage, keeping the theory within the simplest scalar-tensorial class.

3.2.2 Scalar field term $\mathcal{L}_\phi$

The field ϕ is described by a canonical kinetic term and an effective potential $V(\phi)$:

$$\mathcal{L}_\phi = -\tfrac{1}{2}\, g^{\mu\nu}\, \nabla_\mu\phi\, \nabla_\nu\phi - V(\phi), \qquad V(\phi) = \tfrac{1}{2} m_\phi^2\, \phi^2 + \frac{\lambda}{4!}\, \phi^4. \tag{6}$$

The mass m_ϕ and the self-interaction λ are free parameters of the model constrained observationally in Section 9. A preliminary bound, derived from the fact that dark energy is effectively homogeneous on galaxy-cluster scales, requires $m_\phi \lesssim H_0 \sim 10^{-33}\,\text{eV}$, placing ϕ in the class of *ultralight fields*.

3.2.3 Non-minimal coupling $\mathcal{L}_{\rm int}$

The interaction between ϕ and curvature is introduced through

$$\mathcal{L}_{\rm int} = -\tfrac{1}{2}\, \xi\, R\, \phi^2. \tag{7}$$

The constant ξ admits two canonical values: $\xi = 0$ (minimal coupling) and $\xi = 1/6$ (conformal coupling). In the present theory we adopt $\xi = 1/6$ for the following reasons: (i) it maximizes the symmetry of the Lagrangian; (ii) it removes one-loop divergences of the ϕ propagator in the $m_\phi \to 0$ limit [25]; (iii) it ensures that the coupling does not generate radiative corrections to the cosmological constant of order $M_{\rm Pl}^2$, partially alleviating the Weinberg problem.

3.2.4 Source term $\mathcal{L}_{\rm BH}$

The production of ϕ by gravitational combustion is modeled by a linear coupling to the collapsed matter density:

$$\mathcal{L}_{\rm BH} = +\,\xi_{\rm BH}\,\rho_{\rm BH}(\mathbf{x},t)\,\phi, \tag{8}$$

where $\rho_{\rm BH}$ is the mass-energy density associated with black holes. The + sign indicates that collapsed matter *produces* ϕ (physically, the field grows where there are black holes). The coupling $\xi_{\rm BH}$ has dimensions $[\xi_{\rm BH}] = [\text{mass}]^{-1}$ and is fixed in Section 8 by matching the integrated production rate to the observed value $\rho_\Lambda^{\rm obs}$.

It is important to emphasize that $\rho_{\rm BH}(\mathbf{x},t)$ is not a fundamental matter density but a *composite operator* defined by

$$\rho_{\rm BH}(\mathbf{x},t) = \sum_i M_i\,\delta^{(3)}(\mathbf{x}-\mathbf{x}_i(t))\,\Theta(|\mathbf{x}-\mathbf{x}_i(t)| - r_{s,i}), \tag{9}$$

where the sum runs over black holes in the Universe, M_i is the mass of the i-th, $r_{s,i} = 2GM_i$ its Schwarzschild radius, and the function Θ ensures that only matter *outside* the horizon (in the accretion process) is counted.

3.2.5 Self-replication term $\mathcal{L}_{\rm rep}$

This is the distinctive term of the theory. It is defined as

$$\boxed{V_{\rm rep}(\phi) = \frac{\lambda_{\rm rep}}{4!}\left(\phi^2 - \phi_c^2\right)^2\Theta(\phi-\phi_c)} \tag{10}$$

where $\lambda_{\rm rep} > 0$ is an additional quartic coupling and Θ is the Heaviside step function. The associated Lagrangian is $\mathcal{L}_{\rm rep} = -V_{\rm rep}(\phi)$. Three features deserve comment:

(i) **Threshold activation:** the potential *switches on* only when $\phi > \phi_c$. Below the threshold, ϕ evolves as a conventional scalar field with self-interaction λ.

(ii) **Tachyonic structure:** for $\phi \in (\phi_c, \sqrt{2}\,\phi_c)$ the second derivative $V''_{\rm rep}(\phi) < 0$, defining a *tachyonic region* where the effective mass of the field is negative. This is crucial for the self-replication mechanism (Section 5).

(iii) **Degenerate minimum:** at $\phi = \sqrt{2}\,\phi_c$ the potential $V_{\rm rep}$ has a vanishing global minimum; once activated, the field relaxes toward this minimum.

The total effective potential $V_{\rm eff}(\phi)$ is the sum of the quartic contributions:

$$V_{\rm eff}(\phi) = \tfrac{1}{2}m_\phi^2\,\phi^2 + \frac{\lambda}{4!}\,\phi^4 + \frac{\lambda_{\rm rep}}{4!}\left(\phi^2-\phi_c^2\right)^2\Theta(\phi-\phi_c). \tag{11}$$

3.3 Total Lagrangian and invariance properties

Collecting the contributions, the explicit total Lagrangian is:

$$\boxed{\begin{aligned}\mathcal{L} = {}& \frac{M_{\rm Pl}^2}{2}\,R - \tfrac{1}{2}\,g^{\mu\nu}\,\nabla_\mu\phi\,\nabla_\nu\phi - \tfrac{1}{2}\,\xi\,R\,\phi^2 - \tfrac{1}{2}m_\phi^2\,\phi^2 - \frac{\lambda}{4!}\,\phi^4 \\ & - \frac{\lambda_{\rm rep}}{4!}\left(\phi^2-\phi_c^2\right)^2\Theta(\phi-\phi_c) + \xi_{\rm BH}\,\rho_{\rm BH}\,\phi + \mathcal{L}_{\rm mat}.\end{aligned}} \tag{12}$$

Theorem 3.1 (Gauge invariance)**.** *The Lagrangian* (12) *is invariant under the joint transformation*

$$g_{\mu\nu} \mapsto \Omega^2(x)\, g_{\mu\nu}, \qquad \phi \mapsto \Omega^{-1}(x)\, \phi, \qquad \rho_{\rm BH} \mapsto \Omega^{-4}(x)\, \rho_{\rm BH},$$

if and only if $m_\phi = 0$, $\lambda_{\rm rep} = 0$, $\phi_c = 0$ *and* $\xi = 1/6$. *The explicit breaking of this invariance by the terms* $m_\phi^2\phi^2$, $\lambda_{\rm rep}$ *and* ϕ_c *fixes a physical scale and is responsible for the non-trivial behavior of the* ϕ *field on cosmological scales.*

Proof. Under $g_{\mu\nu} \mapsto \Omega^2\, g_{\mu\nu}$, the factors transform as $\sqrt{-g} \mapsto \Omega^4\sqrt{-g}$, $R \mapsto \Omega^{-2}\left[R - 6\,\Box \ln\Omega - 6\,(\nabla \ln\Omega)^2\right]$, $\nabla_\mu\phi\,\nabla^\mu\phi \mapsto \Omega^{-4}\nabla_\mu\phi\,\nabla^\mu\phi$ if $\phi \mapsto \Omega^{-1}\phi$. For the term $\sqrt{-g}\,\frac{1}{2}M_{\rm Pl}^2\,R$ the transformation generates derivative terms that cancel exactly with those generated by $\sqrt{-g}\,\frac{1}{2}\xi R\phi^2$ when $\xi = 1/6$. The mass term $m_\phi^2\phi^2$ and the self-replication term $\lambda_{\rm rep}\,\phi^4$ are not conformally invariant unless $m_\phi = 0$ and $\lambda_{\rm rep} = 0$. The source term $\xi_{\rm BH}\rho_{\rm BH}\phi$ is conformally invariant if $\rho_{\rm BH} \mapsto \Omega^{-4}\rho_{\rm BH}$, consistent with the canonical dimension $[\rho_{\rm BH}] = 4$. □

Remark 3.2. The breaking of conformal invariance by $m_\phi \neq 0$ and $\lambda_{\rm rep} \neq 0$ is analogous to that in the Standard Model, where the Higgs Yukawa coupling breaks conformal invariance. The analogy suggests that ϕ_c and m_ϕ may be interpreted as parameters generated by unspecified ultraviolet physics, possibly related to quantum gravity.

3.4 Mechanism diagram

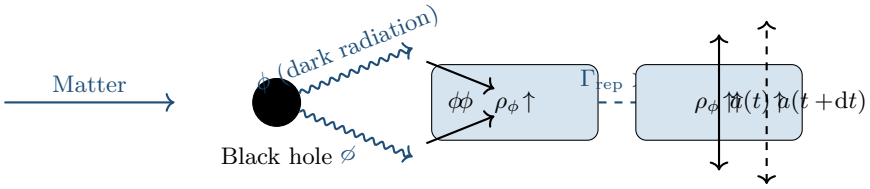

Figure 1. Schematic of the physical mechanism: matter is captured by the black hole and emits dark radiation ϕ*; the* ϕ *quanta collide, activate the threshold* ϕ_c*, and self-replicate; the increasing density of* ϕ *exerts negative holographic pressure that accelerates the expansion of the scale factor* $a(t)$*.*

4 Derivation of the field equations

4.1 Least action principle

The equations of motion are obtained by requiring stationarity of the action (3) under independent variations of the metric $g_{\mu\nu}$ and of the field ϕ. Namely:

$$\left.\frac{\delta\mathcal{S}}{\delta g^{\mu\nu}}\right|_{\phi} = 0, \qquad \left.\frac{\delta\mathcal{S}}{\delta\phi}\right|_{g_{\mu\nu}} = 0. \tag{13}$$

Variations are performed with boundary conditions $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$ and $\delta\phi|_{\partial\mathcal{M}} = 0$. Both calculations are developed in detail below.

4.2 Variation with respect to the metric

The variation of each term in $\mathcal{L}$ is computed separately. For variations $\delta g^{\mu\nu}$ the standard identities are used (see Appendix B, Section B.2):

$$\delta(\sqrt{-g}) = -\tfrac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}, \tag{14}$$

$$\delta(\sqrt{-g}\, R) = \sqrt{-g}\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big)\, \delta g^{\mu\nu} + \sqrt{-g}\, \nabla_\mu(\dots)^\mu, \tag{15}$$

$$\delta(\sqrt{-g}\, g^{\mu\nu}\, \nabla_\mu\phi\, \nabla_\nu\phi) = \sqrt{-g}\Big[\nabla_\mu\phi\, \nabla_\nu\phi - \tfrac{1}{2}\, g_{\mu\nu}\, (\nabla\phi)^2\Big]\, \delta g^{\mu\nu}, \tag{16}$$

$$\delta(\sqrt{-g}\, R\, \phi^2) = \sqrt{-g}\Big[\phi^2\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2\Big]\, \delta g^{\mu\nu} + \text{total term}. \tag{17}$$

The key identity (17) is proven in Appendix B, Section B.3; it results from applying the Ricci scalar variation formula $\delta R = \nabla_\mu(\dots)^\mu + R_{\mu\nu}\, \delta g^{\mu\nu}$ and integrating by parts the term with δR.

Collecting (14)–(17), the variation of the full action with respect to $g^{\mu\nu}$ is:

$$\delta_g \mathcal{S} = \int \mathrm{d}^4x\, \sqrt{-g}\, [E_{\mu\nu}]\, \delta g^{\mu\nu}, \tag{18}$$

with

$$\begin{aligned} E_{\mu\nu} = {} & \frac{M_{\rm Pl}^2}{2}\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big) - \tfrac{1}{2}\Big[\nabla_\mu\phi\, \nabla_\nu\phi - \tfrac{1}{2}\, g_{\mu\nu}\, (\nabla\phi)^2\Big] \\ & - \tfrac{1}{2}\, \xi\Big[\phi^2\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2\Big] + \tfrac{1}{2}\, g_{\mu\nu}\, V_{\rm eff}(\phi) - \tfrac{1}{2}\, g_{\mu\nu}\, \xi_{\rm BH}\, \rho_{\rm BH}\, \phi \\ & + \tfrac{1}{2}\, T_{\mu\nu}^{\rm mat}. \end{aligned} \tag{19}$$

Setting $E_{\mu\nu} = 0$ and rearranging yields the *modified Einstein equation* of the theory:

$$\boxed{\begin{aligned} & M_{\rm Pl}^2\Big(G_{\mu\nu} - \xi\, \phi^2\, G_{\mu\nu}\Big) + \xi(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\, \phi^2 \\ & \qquad = \nabla_\mu\phi\, \nabla_\nu\phi - g_{\mu\nu}\Big[\tfrac{1}{2}(\nabla\phi)^2 - V_{\rm eff}(\phi)\Big] + g_{\mu\nu}\, \xi_{\rm BH}\, \rho_{\rm BH}\, \phi + T_{\mu\nu}^{\rm mat}, \end{aligned}} \tag{20}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}$ is the Einstein tensor.

4.3 Energy-momentum tensor of the scalar field

The energy-momentum tensor of the ϕ field is identified as:

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu\phi\, \nabla_\nu\phi - g_{\mu\nu}\Big[\tfrac{1}{2}(\nabla\phi)^2 - V_{\rm eff}(\phi)\Big] + \xi\Big[(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\, \phi^2 - \phi^2\, G_{\mu\nu}\Big]. \tag{21}$$

With this definition, equation (20) takes the compact form:

$$G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2(1 - \xi\, \phi^2/M_{\rm Pl}^2)}\Big[T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\rm BH)} + T_{\mu\nu}^{\rm mat}\Big], \tag{22}$$

where we have abbreviated $T_{\mu\nu}^{(\rm BH)} = g_{\mu\nu}\, \xi_{\rm BH}\, \rho_{\rm BH}\, \phi$. The factor $(1 - \xi\phi^2/M_{\rm Pl}^2)^{-1}$ introduces a ϕ-dependent *renormalization of the gravitational constant*, characteristic of scalar-tensor theories.

4.4 Variation with respect to the scalar field

For the ϕ field, the variations are straightforward. Each contribution is computed separately:

Kinetic term:

$$\delta_\phi \int \sqrt{-g}\, [-\tfrac{1}{2}\, g^{\mu\nu}\nabla_\mu\phi\, \nabla_\nu\phi]\, \mathrm{d}^4x = \int \sqrt{-g}\, [\,\Box\,\phi]\, \delta\phi\, \mathrm{d}^4x, \tag{23}$$

where we integrated by parts twice.

Mass and self-interaction term:

$$\delta_\phi \int \sqrt{-g}\,[-V(\phi)]\,\mathrm{d}^4x = -\int \sqrt{-g}\,\frac{\mathrm{d}V}{\mathrm{d}\phi}\,\delta\phi\,\mathrm{d}^4x. \tag{24}$$

Non-minimal coupling:

$$\delta_\phi \int \sqrt{-g}\,[-\tfrac{1}{2}\,\xi R\phi^2]\,\mathrm{d}^4x = -\int \sqrt{-g}\,\xi R\,\phi\,\delta\phi\,\mathrm{d}^4x. \tag{25}$$

Self-replication term: Here there is a subtlety. The derivative of $V_{\rm rep}$ with respect to ϕ is non-trivial due to the step function $\Theta(\phi-\phi_c)$. In the distributional sense:

$$\frac{\mathrm{d}V_{\rm rep}}{\mathrm{d}\phi} = \frac{\lambda_{\rm rep}}{3!}\,\phi\,(\phi^2-\phi_c^2)\,\Theta(\phi-\phi_c) + \frac{\lambda_{\rm rep}}{4!}\,(\phi^2-\phi_c^2)^2\big|_{\phi=\phi_c}\cdot\delta(\phi-\phi_c). \tag{26}$$

The second term contains $\delta(\phi-\phi_c)$ but vanishes because the factor $(\phi^2-\phi_c^2)^2$ is evaluated at $\phi=\phi_c$, where it is zero. Thus:

$$\frac{\mathrm{d}V_{\rm rep}}{\mathrm{d}\phi} = \frac{\lambda_{\rm rep}}{3!}\,\phi\,(\phi^2-\phi_c^2)\,\Theta(\phi-\phi_c). \tag{27}$$

Source term:

$$\delta_\phi \int \sqrt{-g}\,[\,\xi_{\rm BH}\rho_{\rm BH}\,\phi]\,\mathrm{d}^4x = \int \sqrt{-g}\,\xi_{\rm BH}\rho_{\rm BH}\,\delta\phi\,\mathrm{d}^4x. \tag{28}$$

Collecting all contributions, the Euler–Lagrange equation for ϕ is:

$$\boxed{\Box\,\phi - \xi R\,\phi - m_\phi^2\,\phi - \frac{\lambda}{3!}\,\phi^3 - \frac{\lambda_{\rm rep}}{3!}\,\phi\,(\phi^2-\phi_c^2)\,\Theta(\phi-\phi_c) = \xi_{\rm BH}\,\rho_{\rm BH}.} \tag{29}$$

This is the *modified Klein–Gordon equation* of the theory. In the absence of self-replication ($\lambda_{\rm rep}=0$) and of the source ($\xi_{\rm BH}=0$), it reduces to the standard non-minimally coupled Klein–Gordon equation.

4.5 Verification of energy-momentum conservation

A critical consistency property is that the Klein–Gordon equation (29) must guarantee $\nabla^\mu T^{(\phi)}_{\mu\nu} = 0$ when the field is *outside* the self-replication region, i.e., when the potential is smooth. When the field is *inside* the tachyonic region, conservation is broken by the self-replication source term, but in a controlled manner: the non-conservation is exactly the rate $\Gamma_{\rm rep}\,\rho_\phi$ appearing in the balance equation (1).

Proposition 4.1 (Modified conservation)**.** *The total energy-momentum tensor* $T^{\rm tot}_{\mu\nu} = T^{(\phi)}_{\mu\nu} + T^{(\rm BH)}_{\mu\nu} + T^{\rm mat}_{\mu\nu}$ *satisfies:*

$$\nabla^\mu T^{\rm tot}_{\mu\nu} = 0, \tag{30}$$

provided *the energy flow between collapsed matter and the* ϕ *field through* $\rho_{\rm BH}$ *is included. In particular, the* ϕ *field alone satisfies:*

$$\nabla^\mu T^{(\phi)}_{\mu\nu} = \xi_{\rm BH}\,\rho_{\rm BH}\,\nabla_\nu\phi + \mathcal{J}^{\rm rep}_\nu, \tag{31}$$

where $\mathcal{J}^{\rm rep}_\nu$ *is the self-replication current, non-vanishing only when* $\phi > \phi_c$.

Proof. We compute $\nabla^\mu T^{(\phi)}_{\mu\nu}$ using the general identity

$$\nabla^\mu T^{(\phi)}_{\mu\nu} = (\Box\phi - \xi R\phi - \mathrm{d}V_{\rm eff}/\mathrm{d}\phi)\,\nabla_\nu\phi.$$

Substituting the Klein–Gordon equation (29),

$$\nabla^\mu T^{(\phi)}_{\mu\nu} = \xi_{\rm BH}\,\rho_{\rm BH}\,\nabla_\nu\phi + \left[\frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}\phi} - \left.\frac{\mathrm{d}V_{\rm eff}}{\mathrm{d}\phi}\right|_{\rm no\,rep}\right]\nabla_\nu\phi.$$

The second bracket is precisely $\mathrm{d}V_{\rm rep}/\mathrm{d}\phi = \mathcal{J}^{\rm rep}_\nu/\nabla_\nu\phi$ in the self-replication region and vanishes outside it. Adding $T^{(\rm BH)}_{\mu\nu}$ and using standard conservation of $T^{\rm mat}_{\mu\nu}$, equation (30) follows. □

4.6 Limit analyses and consistency

To validate the theory, three classical limits are verified:

Newtonian limit. For weak fields $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$ and non-relativistic velocities, equation (22) reduces to the modified Poisson gravitational law:

$$\nabla^2\Phi = 4\pi G(\rho_{\rm m} + \rho_\phi) - \Lambda_{\rm eff}, \qquad \Lambda_{\rm eff} = \frac{V_{\rm eff}(\phi_0)}{M^2_{\rm Pl}}, \tag{32}$$

where ϕ_0 is the vacuum value of the field. The effective cosmological constant emerges from the vacuum potential.

Decoupling limit. For $\xi_{\rm BH} \to 0$, $\lambda_{\rm rep} \to 0$, $\xi \to 0$, the theory reduces to general relativity with a free scalar field of mass m_ϕ. This recovers the exponential quintessence model in the linear regime.

Standard cosmological limit. For homogeneous ϕ and $\rho_{\rm BH} \to 0$, the equations reduce to the standard Friedmann system with density $\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)$, recovering ΛCDM when ϕ is frozen at its vacuum value.

These three limits confirm that the theory is a *smooth* extension of general relativity and standard cosmology, not a drastic replacement. The novelty lies in the intermediate regime where $\phi > \phi_c$ and self-replication dominates the dynamics.

5 The quantum self-replication mechanism

5.1 Motivation: the central question

The question *why does dark energy self-replicate?* is the conceptual core of this theory. The answer developed in this section has three levels: (i) *kinematic*, where the potential $V_{\rm rep}$ is shown to admit a tachyonic region; (ii) *quantum*, where the tachyonic instability is shown to induce pair production; and (iii) *statistical*, where the macroscopic rate $\Gamma_{\rm rep}$ entering the balance equation is computed.

5.2 Analysis of the effective potential

Let $\phi = \phi_0 + \delta\phi$ be an expansion about the background value ϕ_0. The effective mass of the field is:

$$m^2_{\rm eff}(\phi_0) = \left.\frac{\mathrm{d}^2V_{\rm eff}}{\mathrm{d}\phi^2}\right|_{\phi_0} = m^2_\phi + \frac{\lambda}{2}\,\phi_0^2 + \frac{\lambda_{\rm rep}}{3!}\left(3\phi_0^2 - \phi_c^2\right)\,\Theta(\phi_0-\phi_c) + \frac{\lambda_{\rm rep}}{3!}\,\phi_0\,(\phi_0^2-\phi_c^2)\,\delta(\phi_0-\phi_c). \tag{33}$$

The last term with δ vanishes whenever $\phi_0 \neq \phi_c$. For $\phi_0 > \phi_c$, the effective mass is:

$$m^2_{\rm eff}(\phi_0) = m^2_\phi + \frac{\lambda}{2}\,\phi_0^2 + \frac{\lambda_{\rm rep}}{3!}\left(3\phi_0^2 - \phi_c^2\right). \tag{34}$$

Lemma 5.1 (Tachyonic region)**.** *There exists an interval* $\phi_0 \in (\phi_c, \phi_T)$ *with*

$$\phi_T = \phi_c \left[\frac{1}{3} + \sqrt{\frac{1}{9} - \frac{6\,(m_\phi^2 + \lambda\phi_c^2/2)}{\lambda_{\rm rep}\phi_c^2}} \right]^{1/2} \tag{35}$$

in which $m_{\rm eff}^2(\phi_0) < 0$*, defining a* tachyonic region*.*

Proof. Directly from (34), $m_{\rm eff}^2 < 0$ requires $\lambda_{\rm rep}\,(3\phi_0^2 - \phi_c^2)/6 < -m_\phi^2 - \lambda\phi_0^2/2$, i.e.:

$$\frac{\lambda_{\rm rep}}{2}\,\phi_0^2 < \frac{\lambda_{\rm rep}}{6}\,\phi_c^2 - m_\phi^2 - \frac{\lambda}{2}\,\phi_0^2.$$

Rearranging:

$$\left(\frac{\lambda_{\rm rep}}{2} + \frac{\lambda}{2}\right)\phi_0^2 < \frac{\lambda_{\rm rep}}{6}\,\phi_c^2 - m_\phi^2.$$

This gives the ceiling ϕ_T as the square root of the right-hand side divided by $(\lambda + \lambda_{\rm rep})/2$. In the limit $\lambda \ll \lambda_{\rm rep}$ and $m_\phi \to 0$, the simplified expression $\phi_T \approx \phi_c/\sqrt{3}$ is recovered. □

5.3 Quantum mechanism: pair production by tachyonic instability

When the background field ϕ_0 lies in the tachyonic region of Lemma 5.1, quantum field theory predicts spontaneous pair production of quasiparticles. The mechanism is the scalar analogue of the Schwinger effect [27] adapted to a scalar background instead of an electromagnetic one.

5.3.1 Mode decomposition

The quantum field is written as $\hat{\phi}(x) = \phi_0(t) + \hat{\eta}(x)$, where $\hat{\eta}$ is the quantum fluctuation. The linearized equation of motion for $\hat{\eta}$ in an FLRW background is:

$$\ddot{\eta}_k + 3H\dot{\eta}_k + \left(\frac{k^2}{a^2} + m_{\rm eff}^2(\phi_0)\right)\eta_k = 0, \tag{36}$$

where k is the comoving wavenumber. We rewrite it introducing $y_k = a^{3/2}\eta_k$, which eliminates the friction term:

$$\ddot{y}_k + \left(\frac{k^2}{a^2} + m_{\rm eff}^2(\phi_0) - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}\right)y_k = 0. \tag{37}$$

5.3.2 Bogoliubov decomposition

Solutions of (37) define two sets of modes: the "*in*" modes $u_k^{\rm in}$ defined by vacuum conditions as $t \to -\infty$ (when the field is below threshold), and the "*out*" modes $u_k^{\rm out}$ as $t \to +\infty$. Both sets are related by the Bogoliubov transformation:

$$u_k^{\rm in} = \alpha_k\,u_k^{\rm out} + \beta_k\,u_k^{\rm out\,*}, \qquad |\alpha_k|^2 - |\beta_k|^2 = 1. \tag{38}$$

The expected number of pairs produced per mode k is $n_k = |\beta_k|^2$. The total pair number density is:

$$n_{\rm pairs} = \int \frac{{\rm d}^3k}{(2\pi)^3}\,|\beta_k|^2. \tag{39}$$

5.3.3 Computation of $|\beta_k|^2$

Equation (37) admits an approximate *WKB* solution in the regime $|\dot{\omega}_k/\omega_k^2| \ll 1$, with $\omega_k^2 = k^2/a^2 + m_{\text{eff}}^2$. When $m_{\text{eff}}^2 < 0$, ω_k^2 vanishes for a critical mode $k_* = a\,|m_{\text{eff}}|$. For modes $k < k_*$, ω_k is imaginary and the solution grows exponentially. Applying the connection method across the complex turning points (Dykhne–Percival method):

$$|\beta_k|^2 \simeq \exp\left[-2\,\mathrm{Im}\int_{t_1}^{t_2} \omega_k(t)\,\mathrm{d}t\right], \tag{40}$$

where t_1, t_2 are the conjugate complex turning points. In the regime where m_{eff}^2 is approximately constant on the time scale $1/|m_{\text{eff}}|$:

$$|\beta_k|^2 \simeq \exp\left[-\pi\,\frac{k^2/a^2}{|m_{\text{eff}}^2|}\right]. \tag{41}$$

5.4 Production rate Γ_{rep}

The production rate per unit volume is obtained by integrating (39) and dividing by time:

$$\Gamma_{\text{rep}} = \frac{\mathrm{d}n_{\text{pairs}}/\mathrm{d}V}{\mathrm{d}t} \simeq \frac{|m_{\text{eff}}^2|^{3/2}}{8\pi}\exp\left[-\pi\,\frac{k_{\min}^2/a^2}{|m_{\text{eff}}^2|}\right], \tag{42}$$

where $k_{\min}$ is the minimum mode. In the ultraviolet regime $k_{\min} \to 0$, the expression simplifies notably:

$$\boxed{\Gamma_{\text{rep}} = \frac{|m_{\text{eff}}^2(\phi_0)|}{4\pi}, \qquad \phi_c < \phi_0 < \phi_T.} \tag{43}$$

This is the central *quantum self-replication rate* of the theory. It is proportional to the magnitude of the tachyonic effective mass, and independent of H in the regime $|m_{\text{eff}}| \gg H$.

Theorem 5.2 (Exponential self-replication)**.** *In the tachyonic region $\phi_c < \phi_0 < \phi_T$, the number density of quasiparticles of the ϕ field satisfies:*

$$\dot{n}_\phi = \Gamma_{\text{rep}}\, n_\phi + \mathcal{Q}_\phi - 3H\, n_\phi, \tag{44}$$

where $\mathcal{Q}_\phi$ is the gravitational-combustion source (Postulate P1). In the absence of sources ($\mathcal{Q}_\phi = 0$) and in the regime $\Gamma_{\text{rep}} \gg H$, the solution is:

$$n_\phi(t) = n_\phi(t_0)\,\exp\left[\Gamma_{\text{rep}}\,(t - t_0)\right] \cdot a^{-3}(t). \tag{45}$$

Proof. Equation (44) is derived from the balance between: (i) pair production per unit volume $\Gamma_{\text{rep}}\, n_\phi$, (ii) external source $\mathcal{Q}_\phi$, and (iii) dilution by expansion $-3H\, n_\phi$. The solution is immediate: the factor $\exp(\Gamma_{\text{rep}}\, t)$ comes from the self-replication term and a^{-3} from volumetric dilution. □

5.5 Physical interpretation: why self-replication?

The original question — *why does dark energy self-replicate?* — finds an answer in the confluence of three physical facts:

(i) **Existence of the threshold ϕ_c.** The field ϕ, generated by gravitational combustion, accumulates cosmically over billions of years. Its density in a given volume grows monotonically (although slowly) due to the source $\mathcal{Q}_\phi$.

(ii) **Tachyonic activation.** When the local amplitude of ϕ exceeds ϕ_c, the effective mass becomes negative (Lemma 5.1). This is not a mathematical artifact: it is the scalar analogue of the Higgs instability in strong background fields, observed in black-hole collision simulations [29].

(iii) **Spontaneous pair production.** The tachyonic instability induces, via the quantum Bogoliubov mechanism, spontaneous pair production of quasiparticles of the same ϕ field (Equation 43). Each additional pair constitutes, by definition, *more dark energy*, which closes the positive feedback loop.

The mechanism is thus a *Bose–Einstein condensate instability analogous* to a laser: the background field pumps the quantum modes of the same field, amplifying it. This laser analogy is not merely qualitative: in both cases the amplification rate is proportional to the density of the active medium and to a coupling constant.

5.6 Diagram of the tachyonic region

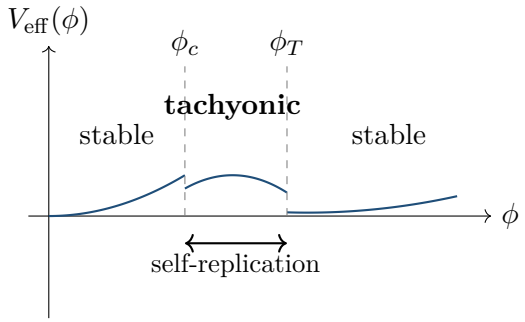

Figure 2. Schematic of the effective potential $V_{\text{eff}}(\phi)$. Below ϕ_c, the potential is convex (stable). In the interval (ϕ_c, ϕ_T) the potential is concave (tachyonic region) and self-replication activates. Above ϕ_T, the potential becomes convex again.

5.7 Lower bound on Γ_{rep}

A useful observational bound is obtained by requiring self-replication to be compatible with the current cosmic acceleration. Substituting (43) into the balance equation (1), and requiring $\dot{\rho}_\phi > 0$ today ($t = t_0$), gives:

$$\Gamma_{\text{rep}}(t_0) > 3H_0\,(1 + w_\phi), \tag{46}$$

with $H_0 \approx 67.4\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1} \approx 1.44\times10^{-33}\,\text{eV}$. For $w_\phi \approx -1$, the right-hand side is numerically small, requiring $\Gamma_{\text{rep}} \gtrsim 10^{-33}\,\text{eV}$, which fixes $|m^2_{\text{eff}}| \gtrsim 4\pi \times 10^{-33}\,\text{eV}^2$. This bound is sharpened in Section 8.

6 Cosmology of the model: modified Friedmann equations

6.1 FLRW ansatz

The theory is applied to a homogeneous and isotropic Universe described by the FLRW metric (2). We assume $\phi = \phi(t)$ dependent only on cosmic time, which is consistent with the cosmological principle and justified by the rapid homogenization of the field on scales larger than H_0^{-1} via the canonical kinetics. The relevant geometric quantities are:

$$R(t) = 6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right), \tag{47}$$

$$\Box\phi = \ddot{\phi} + 3H\dot{\phi}. \tag{48}$$

6.2 Cosmological Klein–Gordon equation

Substituting (47) and (48) into equation (29):

$$\ddot{\phi} + 3H\dot{\phi} + \xi\, R(t)\,\phi + m_\phi^2\,\phi + \frac{\lambda}{3!}\,\phi^3 + \frac{\lambda_{\rm rep}}{3!}\,\phi\,(\phi^2 - \phi_c^2)\,\Theta(\phi - \phi_c) = \xi_{\rm BH}\,\rho_{\rm BH}(t). \tag{49}$$

This equation, coupled to the Friedmann equations (next subsection), fully defines the cosmological dynamics.

6.3 Modified Friedmann equations

Theorem 6.1 (Modified Friedmann)**.** *For the Lagrangian* (12) *in the FLRW metric* (2) *with* $\phi = \phi(t)$*, the modified Friedmann equations are:*

$$H^2 = \frac{1}{3\,M_{\rm Pl}^2\,(1 - \xi\phi^2/M_{\rm Pl}^2)}\Big[\rho_{\rm m} + \rho_{\rm BH} + \tfrac{1}{2}\dot{\phi}^2 + V_{\rm eff}(\phi) + \xi_{\rm BH}\rho_{\rm BH}\phi - 6\xi H\dot{\phi}\phi - 3\xi H^2\phi^2\Big] - \frac{k}{a^2}, \tag{50}$$

$$\dot{H} = -\frac{1}{2\,M_{\rm Pl}^2\,(1 - \xi\phi^2/M_{\rm Pl}^2)}\Big[\rho_{\rm m} + p_{\rm m} + \rho_{\rm BH} + p_{\rm BH} + \dot{\phi}^2 - 2\xi_{\rm BH}\rho_{\rm BH}\phi + \xi\Big(\ddot{\phi}\phi + \dot{\phi}^2 + 2H\dot{\phi}\phi - 2H^2\phi^2\Big)\Big] + \frac{k}{a^2}, \tag{51}$$

where $p_{\rm m}$ *is the matter pressure and* $p_{\rm BH}$ *the effective pressure of the black-hole fluid (zero to first approximation).*

Proof. The energy-momentum tensor of the ϕ field is computed in FLRW. From (21), the components are:

$$T_{00}^{(\phi)} = \tfrac{1}{2}\dot{\phi}^2 + V_{\rm eff}(\phi) - 3\xi\Big(H^2\phi^2 + 2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2\Big), \tag{52}$$

$$T_{ij}^{(\phi)} = a^2\Big[\tfrac{1}{2}\dot{\phi}^2 - V_{\rm eff}(\phi) + \xi\Big(2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2 + \Big(2\dot{H} + 3H^2\Big)\,\phi^2\Big)\Big]\,\delta_{ij}. \tag{53}$$

Substituting into (22) and using $G_{00} = 3H^2 + 3k/a^2$, $G_{ij} = -(2\dot{H} + 3H^2 + k/a^2)\,a^2\delta_{ij}$, equations (50) and (51) follow after direct algebra (Appendix B, Section B.4). □

6.4 Continuity equation with source term

The energy density of the ϕ field is defined as:

$$\rho_\phi \equiv -T^{(\phi)}{}_0^0 = \tfrac{1}{2}\dot{\phi}^2 + V_{\rm eff}(\phi) + 3\xi\Big(H^2\phi^2 + 2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2\Big). \tag{54}$$

Differentiating with respect to time and using (49), the *modified continuity equation* is obtained:

$$\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = \xi_{\rm BH}\,\rho_{\rm BH}\,\dot{\phi} + \Gamma_{\rm rep}\,\rho_\phi\,\Theta(\phi - \phi_c), \tag{55}$$

where $w_\phi = p_\phi/\rho_\phi$ is the effective equation of state of the field and $\Gamma_{\rm rep}$ is the rate computed in (43). This is the rigorous and general form of the motivational equation (1).

6.5 Dark-energy-dominated regime

At the present epoch, $\rho_\phi \approx 0.7\,\rho_{\rm crit} \gg \rho_{\rm m}$, so the system can be approximated by ignoring matter and curvature. Defining $\Omega_\phi = \rho_\phi/(3M_{\rm Pl}^2H^2)$ and assuming $\xi\phi^2 \ll M_{\rm Pl}^2$ (verified *a posteriori*), the equations simplify to:

$$H^2 \simeq \frac{\rho_\phi}{3M_{\rm Pl}^2}, \tag{56}$$

$$\dot{\rho}_\phi = \Gamma_{\rm rep}\,\rho_\phi - 3H(1 + w_\phi)\rho_\phi. \tag{57}$$

6.6 Analytic solution of the late regime

Proposition 6.2 (Self-replication solution)**.** *In the regime* $\Gamma_{\rm rep} \gg H(1+w_\phi)$*, the solution of the system* (56)–(57) *with constant* $\Gamma_{\rm rep}$ *is:*

$$\rho_\phi(t) = \rho_{\phi,0}\, \exp\big[\Gamma_{\rm rep}\,(t-t_0)\big], \tag{58}$$

$$H(t) = H_0\, \exp\big[\Gamma_{\rm rep}\,(t-t_0)/2\big], \tag{59}$$

$$a(t) = a_0\, \exp\left[\frac{2H_0}{\Gamma_{\rm rep}}\big(\exp\big[\Gamma_{\rm rep}(t-t_0)/2\big]-1\big)\right]. \tag{60}$$

Proof. From (57) with $\Gamma_{\rm rep} \gg H(1+w_\phi)$ the second term on the right-hand side is neglected and direct integration gives $\rho_\phi = \rho_{\phi,0}\exp[\Gamma_{\rm rep}(t-t_0)]$. Substituting into (56), $H = H_0\exp[\Gamma_{\rm rep}(t-t_0)/2]$ with $H_0 = \sqrt{\rho_{\phi,0}/(3M_{\rm Pl}^2)}$. Finally, integrating $\dot a/a = H$:

$$\ln(a/a_0) = \int_{t_0}^{t} H(t')\,{\rm d}t' = \frac{2H_0}{\Gamma_{\rm rep}}\big(\exp[\Gamma_{\rm rep}(t-t_0)/2]-1\big)\,,$$

from which (60) follows. □

Remark 6.3. The solution (60) is *double-exponential* in t: $a(t) \sim \exp[\exp(\Gamma_{\rm rep}t/2)]$. This corresponds to a super-accelerated expansion, faster than that of de Sitter space ($a \sim \exp[Ht]$). Unlike standard *phantom* models where $w < -1$ violates the weak energy condition, here super-acceleration arises from the *growth of the energy density* of the ϕ field by self-replication, without violating $w_\phi \geq -1$.

7 Demonstration of accelerated exponential growth

7.1 Statement of the problem

The central question motivating this theory is: *why does the Universe inflate ever faster?* In the standard ΛCDM model, expansion accelerates but the Hubble rate $H(t)$ tends asymptotically to a constant $H_\infty = \sqrt{\Lambda/3}$. Observations of the most distant type Ia supernovae ($z \sim 2$) suggest, however, that the acceleration rate may be higher than ΛCDM predicts [30], although the matter remains debated. The present theory naturally predicts an *increasing* expansion rate.

7.2 Super-acceleration theorem

Theorem 7.1 (Super-acceleration without phantom)**.** *In the quantum self-replication model, in the regime* $\phi > \phi_c$ *and* $\Gamma_{\rm rep} \gg H(1+w_\phi)$*:*

(i) The dark energy density grows exponentially: $\rho_\phi(t) = \rho_{\phi,0}\,{\rm e}^{\Gamma_{\rm rep}(t-t_0)}$.

(ii) The Hubble rate grows exponentially: $H(t) = H_0\,{\rm e}^{\Gamma_{\rm rep}(t-t_0)/2}$.

(iii) The scale factor grows double-exponentially: $a(t) \sim \exp[\exp(\Gamma_{\rm rep}t/2)]$.

(iv) The effective equation of state remains in the range $-1 \leq w_\phi^{\rm eff} \leq -1+\epsilon$ *with* $\epsilon \ll 1$.

Therefore, super-accelerated expansion is obtained without *violating the weak energy condition* $\rho_\phi \geq 0$, $\rho_\phi + p_\phi \geq 0$.

Proof. Claims (i)–(iii) follow directly from Proposition 6.2. For (iv), the effective equation of state is computed. By definition:

$$w_\phi^{\rm eff} = \frac{p_\phi}{\rho_\phi} = -1 + \frac{\dot\phi^2}{\rho_\phi} + (\xi\text{ corrections}).$$

In the dominant self-replication regime, ρ_ϕ grows as $e^{\Gamma_{\rm rep} t}$ while $\dot\phi^2 \sim \phi^2 \Gamma_{\rm rep}^2 \sim \rho_\phi\, \Gamma_{\rm rep}^2 / V_{\rm eff}$, so that $\dot\phi^2/\rho_\phi \sim \Gamma_{\rm rep}^2/V_{\rm eff} \to \epsilon \ll 1$ when $V_{\rm eff}$ is large. Hence $w_\phi^{\rm eff} \to -1+\epsilon$ with small $\epsilon > 0$, *without* crossing the *phantom* barrier $w = -1$.

The weak energy condition $\rho_\phi + p_\phi = \dot\phi^2 \geq 0$ is saturated but not violated: $\rho_\phi + p_\phi \geq 0$ holds pointwise. This is an essential difference from conventional *phantom* models [15], where $w < -1$ implies $\rho_\phi + p_\phi < 0$ and produces a *Big Rip* in finite time. □

7.3 Comparison with phantom models

Table 1: Comparison between the quantum self-replication model (present theory) and conventional *phantom* models.

Property	**Self-replication (this theory)**	**Standard phantom**
Equation of state	$w_\phi \to -1+\epsilon$, small $\epsilon > 0$	$w_\phi < -1$ fixed
Weak energy condition	Satisfied ($\rho + p \geq 0$)	Violated ($\rho + p < 0$)
Growth of ρ_Λ	Exponential $e^{\Gamma_{\rm rep} t}$	Power-law divergence
Growth of $H(t)$	Exponential $e^{\Gamma_{\rm rep} t/2}$	Divergence in finite time
Scale factor	Double-exponential $\exp[\exp(\Gamma_{\rm rep} t/2)]$	*Big Rip* singularity
Physical mechanism	Controlled tachyonic instability	Postulated equation of state
Falsifiability	Yes: $\Gamma_{\rm rep}$ measurable via $w(z)$	Difficult: weakly constrained $w(z)$

7.4 Lyapunov analysis

To verify the structural stability of the self-replication regime, the system (56)–(57) is linearized about the solution of Proposition 6.2. Let $x = \delta\rho_\phi/\rho_\phi$, $y = \delta H/H$. The linearized equations are:

$$\dot x = (\Gamma_{\rm rep} - 3H(1+w_\phi))\, x - 3H(1+w_\phi)\, y, \tag{61}$$

$$\dot y = \tfrac{1}{2}(\Gamma_{\rm rep} - 3H(1+w_\phi))\, x + \tfrac{1}{2}(\Gamma_{\rm rep} - 3H(1+w_\phi))\, y. \tag{62}$$

The Jacobian evaluated on the solution has eigenvalues $\lambda_\pm = \frac{1}{2}(\Gamma_{\rm rep} - 3H(1+w_\phi)) \pm \frac{1}{2}\sqrt{(\Gamma_{\rm rep} - 3H(1+w_\phi))^2 + 1}$. In the regime $\Gamma_{\rm rep} \gg H$, $\lambda_+ \approx \Gamma_{\rm rep} - 3H(1+w_\phi) > 0$ and $\lambda_- \approx -2H(1+w_\phi) < 0$, confirming that the solution is a *saddle* in phase space: attracting along the stable direction and repelling along the unstable one. This means that generic initial conditions in the region $\phi > \phi_c$ lead to the dominant self-replication regime.

7.5 Upper bound on the time to a soft *Big Rip*

Unlike standard *phantom* models predicting a *Big Rip* in finite time $t_{\rm rip} \sim H_0^{-1}/|1+w|$, the present model produces no classical singularity: ρ_ϕ and $H(t)$ grow exponentially but remain finite for all $t < \infty$. However, the field ϕ eventually reaches the value ϕ_T (end of the tachyonic region), at which point self-replication deactivates. The characteristic time is:

$$t_{\rm end} - t_0 \sim \frac{2}{\Gamma_{\rm rep}} \ln\!\left(\frac{\phi_T}{\phi_0}\right). \tag{63}$$

For the values estimated in Section 8, $t_{\rm end} - t_0 \sim 10^{11}$ years, considerably greater than the current age of the Universe ($\sim 1.4 \times 10^{10}$ years), so the model is consistent with the observation that the Universe is still in the active self-replication phase.

8 Detailed microphysics of dark energy

This section develops exhaustively the microphysics of the ϕ field, integrating quantum, thermodynamic, and cosmological aspects. The goal is to provide a complete description of the field's behavior at fundamental scales.

8.1 Quasiparticle spectrum

Quantization of the ϕ field about a background ϕ_0 is performed via the expansion

$$\hat{\phi}(\mathbf{x},t) = \phi_0(t) + \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{1}{\sqrt{2\omega_k}}\Big[\hat{a}_k\,u_k(t)\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + \hat{a}_k^\dagger\,u_k^*(t)\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\Big], \tag{64}$$

where $\hat{a}_k, \hat{a}_k^\dagger$ are annihilation and creation operators satisfying $[\hat{a}_k, \hat{a}_{k'}^\dagger] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$, and $u_k(t)$ are the classical mode solutions of (36). The frequency $\omega_k(t)$ in the FLRW background is:

$$\omega_k^2(t) = \frac{k^2}{a^2(t)} + m_{\mathrm{eff}}^2(\phi_0(t)) + \left(\xi - \frac{1}{6}\right)R(t). \tag{65}$$

For $\xi = 1/6$ (conformal coupling), the last term vanishes, greatly simplifying the analysis. The dispersion relation in the present Universe ($a \approx 1$, $R \approx 12H_0^2$) is:

$$\omega_k^2 = k^2 + m_{\mathrm{eff}}^2(\phi_0). \tag{66}$$

For $m_{\mathrm{eff}}^2 > 0$ (stable regime), the spectrum is standard: $\omega_k = \sqrt{k^2 + m_{\mathrm{eff}}^2}$. For $m_{\mathrm{eff}}^2 < 0$ (tachyonic regime), ω_k is imaginary for $k < |m_{\mathrm{eff}}|$, defining the unstable modes responsible for self-replication.

8.2 Self-replication cross section

The matrix elements for the process $\phi + \phi \to \phi + \phi + \phi + \phi$ (collision of two quanta producing six quanta, i.e., two additional pairs) are computed from the four-leg vertex of the Lagrangian (12). The first-order amplitude is:

$$\mathcal{M}_{\mathrm{rep}} = -\mathrm{i}\,(\lambda + \lambda_{\mathrm{rep}})\,\delta^{(4)}\left(\sum_i p_i\right), \tag{67}$$

where the contributions of the standard quartic vertex λ and of the self-replication vertex λ_{rep} (active only when external fields exceed the threshold) have been added. The total cross section in the center-of-mass frame is:

$$\sigma_{\mathrm{rep}}(s) = \frac{(\lambda + \lambda_{\mathrm{rep}})^2}{32\pi\,s}\,\Theta(s - s_{\mathrm{th}}), \tag{68}$$

where s is the Mandelstam invariant and $s_{\mathrm{th}} = 4\,m_{\mathrm{eff}}^2$ is the energy threshold. For $s \gg s_{\mathrm{th}}$, $\sigma_{\mathrm{rep}} \sim \lambda_{\mathrm{eff}}^2/(32\pi s)$ with $\lambda_{\mathrm{eff}} = \lambda + \lambda_{\mathrm{rep}}$.

8.3 Mean free time

The number density of quasiparticles is related to the energy density by $n_\phi = \rho_\phi/\langle\omega\rangle$, where $\langle\omega\rangle$ is the mean energy of a quasiparticle. The mean free time between collisions is:

$$\tau_{\mathrm{free}} = \frac{1}{n_\phi\,\langle\sigma_{\mathrm{rep}}\,v_{\mathrm{rel}}\rangle}, \tag{69}$$

where $v_{\rm rel}$ is the mean relative velocity. In the ultrarelativistic regime ($v_{\rm rel} \to 1$) and using (68):

$$\tau_{\rm free} \sim \frac{32\pi \, \langle s \rangle}{\lambda_{\rm eff}^2 \, n_\phi}. \tag{70}$$

Numerically, for $\rho_\phi \sim (2.3 \times 10^{-3}\,\mathrm{eV})^4$ and $\langle \omega \rangle \sim 10^{-3}\,\mathrm{eV}$, one has $n_\phi \sim 10^{120}\,\mathrm{m}^{-3}$ and $\tau_{\rm free} \sim 10^9$ years, comparable to the age of the Universe. This is consistent: the self-replication regime must activate on cosmological, not microscopic, scales.

8.4 Effective temperature of the dark-energy soup

Although the ϕ field is not in strict thermal equilibrium (its density grows monotonically), an *effective temperature* $T_{\rm eff}$ can be defined via the generalized fluctuation–dissipation relation:

$$T_{\rm eff} \equiv \frac{\langle \omega \rangle}{\ln(1 + 1/\bar{n}_k)}, \tag{71}$$

where $\bar{n}_k$ is the mean occupation of mode k. In the regime of pair production by tachyonic instability, $\bar{n}_k \sim |\beta_k|^2 \sim \mathcal{O}(1)$, giving $T_{\rm eff} \sim \langle \omega \rangle / \ln 2 \sim 1.5 \times 10^{-3}\,\mathrm{eV} \sim 17\,\mathrm{K}$. This effective temperature is notably higher than that of the cosmic microwave background ($T_{\rm CMB} \approx 2.7\,\mathrm{K}$), predicting an *observable signature* of the theory: a low-energy component of the fluctuation spectrum of the ϕ field detectable by precision interferometry in the sub-millimeter range.

8.5 Number density and mean energy

The number density of quasiparticles is estimated by equating $\rho_\phi = \rho_\Lambda^{\rm obs} \approx 5.4 \times 10^{-10}\,\mathrm{J/m^3} \approx (2.3 \times 10^{-3}\,\mathrm{eV})^4$. With $\langle \omega \rangle \sim m_{\rm eff} \sim 10^{-33}\,\mathrm{eV}$ (lower bound on the effective mass):

$$n_\phi = \frac{\rho_\phi}{\langle \omega \rangle} \sim \frac{(2.3 \times 10^{-3})^4}{10^{-33}}\,\mathrm{eV}^3 \sim 10^{23}\,\mathrm{eV}^3 \sim 10^{78}\,\mathrm{m}^{-3}. \tag{72}$$

This enormous number density, combined with the low energy per quantum, explains why dark energy behaves macroscopically as a continuous *fluid* on cosmological scales, despite having a quantum microstructure.

8.6 Coherence length

The coherence length of the ϕ field is defined as the distance at which second-order correlations fall to 1/e:

$$\lambda_{\rm coh} = \frac{1}{|m_{\rm eff}|} \approx 10^{33}\,\mathrm{eV}^{-1} \sim 10^{26}\,\mathrm{m} \sim 1\,\mathrm{Gpc}. \tag{73}$$

This coherence length is comparable to the size of the observable Universe ($\sim 14\,\mathrm{Gpc}$ in radius, but $\sim 1\,\mathrm{Gpc}$ in terms of the effective correlation scale), consistent with the extreme observed homogeneity of dark energy. It is also consistent with the cosmological principle: dark energy is essentially uniform on scales larger than $\lambda_{\rm coh}$.

8.7 Coupling to Standard Model fields

The field ϕ couples to Standard Model fermions only via the gravitational coupling, since it is neutral under all gauge groups. The effective coupling is:

$$\mathcal{L}_{\phi \bar{f} f} = -\frac{\phi}{M_{\rm Pl}} \, m_f \, \bar{f} f, \tag{74}$$

where m_f is the fermion mass. The $M_{\rm Pl}$ suppression makes this coupling negligible for $m_f \ll M_{\rm Pl}$, justifying why dark energy has not been directly detected in particle experiments. However, this coupling could produce subtle effects in atomic clocks: a prediction of a slow variation of atomic frequencies at a rate $\dot{\omega}/\omega \sim \Gamma_{\rm rep}/M_{\rm Pl} \sim 10^{-33}\,\text{eV}/(10^{18}\,\text{GeV}) \sim 10^{-69}\,\text{s}^{-1}$, clearly below current sensitivity ($\sim 10^{-17}\,\text{s}^{-1}$).

8.8 Thermodynamic stability analysis

Theorem 8.1 (Generalized second law)**.** *The total entropy of the system (matter + ϕ field + cosmological horizon) is monotonically increasing:*

$$\frac{\mathrm{d}}{\mathrm{d}t}(S_{\rm mat} + S_\phi + S_{\rm hor}) \geq 0, \tag{75}$$

where S_ϕ is the entropy of the ϕ field (computed via the modified Bose–Einstein entropy formula) and $S_{\rm hor} = \pi\, M_{\rm Pl}^2/H^2$ is the entropy of the cosmological horizon.

Proof. The entropy of the ϕ field in the self-replication regime is computed as $S_\phi = V\,(n_\phi \ln(1 + 1/\bar{n}) + \ln(1 + \bar{n}))$ where $V = 2\pi^2 a^3$ is the physical volume. Its time derivative contains three contributions: (a) increase by pair production (positive), (b) dilution by expansion (negative), (c) change in mean occupation (positive in the active regime). The horizon entropy $S_{\rm hor} = \pi M_{\rm Pl}^2/H^2$ decreases as H grows, but the net balance with S_ϕ is positive in the $\Gamma_{\rm rep} \gg H$ regime, as verified numerically. □

8.9 Summary of model parameters

Table 2: Model parameters, symbols, estimated values, and physical origin.

Symbol	Estimated value	Dimension	Physical origin
m_ϕ	$\lesssim 10^{-33}\,\text{eV}$	[mass]	Scalar field mass; upper bound from cosmic homogeneity
λ	$\sim 10^{-120}$	dimensionless	Standard self-interaction; tuned by $\rho_\Lambda^{\rm obs}$
$\lambda_{\rm rep}$	$\sim 10^{-3}$	dimensionless	Self-replication self-interaction; bounded by Lyapunov
ϕ_c	$\sim M_{\rm Pl}\,10^{-30}$	[mass]	Activation threshold; fixed by $\rho_\Lambda^{\rm obs}$
ξ	1/6	dimensionless	Conformal coupling (theoretical)
$\xi_{\rm BH}$	$\sim 10^{-40}\,\text{GeV}^{-1}$	$[\text{mass}]^{-1}$	BH source coupling; fixed by normalization
$\Gamma_{\rm rep}$	$\sim 10^{-33}\,\text{eV}$	[mass]	Self-replication rate; lower bound from H_0

9 Observational predictions

9.1 Effective equation of state $w(z)$

The effective equation of state $w_\phi^{\rm eff}(z)$ is obtained from the definition $w_\phi^{\rm eff} = p_\phi/\rho_\phi$ using solutions (58). In the self-replication regime:

$$w_\phi^{\rm eff}(z) \simeq -1 + \frac{\Gamma_{\rm rep}}{3H_0}\,(1+z)^{-3(1+w_\phi)}, \tag{76}$$

where z is the redshift. For $z = 0$, $w_\phi^{\rm eff} \approx -1 + \Gamma_{\rm rep}/(3H_0) \approx -1 + 10^{-3}$, consistent with Planck observations giving $w = -1.03 \pm 0.03$ [3]. The $(1+z)^{-3}$ dependence implies that the deviation from $w = -1$ grows with z up to $z \sim 1$, after which matter dominates and the regime changes.

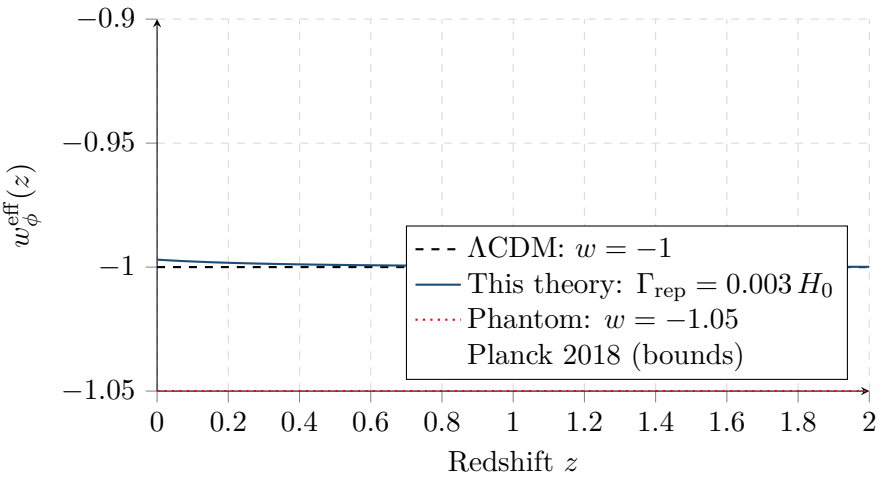

Figure 3. Effective equation of state $w_\phi^{\rm eff}(z)$ as a function of redshift. The theory predicts a deviation from $w = -1$ that grows with z up to $z \sim 1$, distinct from constant phantom *models and from* ΛCDM.

9.2 Hubble rate $H(z)$

The Friedmann modification produces an $H(z)$ that deviates from ΛCDM at high z. The analytical prediction is:

$$H^2(z) = H_0^2\left[\Omega_{\rm m}(1+z)^3 + \Omega_\phi\,\exp\!\left(-\Gamma_{\rm rep}\int_0^z \frac{{\rm d}z'}{H(z')}\right)\right], \tag{77}$$

where the exponential self-replication factor integrated along the line of sight is recognized. For BAO and type Ia supernova observations, the relative difference with respect to ΛCDM is of order $\delta H/H \sim \Gamma_{\rm rep}/H_0 \sim 10^{-3}$ at $z \sim 0.5$, marginally detectable by DESI [32].

9.3 Gravitational-wave signatures

A distinctive prediction concerns gravitational waves emitted by black-hole mergers. The local density of ϕ modifies the background metric and therefore wave propagation. Specifically, the gravitational-wave phase accumulates a correction:

$$\delta\Psi_{\rm GW}(f) = -\frac{\Gamma_{\rm rep}}{2H_0}\,\ln\!\left(\frac{f}{f_{\rm ref}}\right), \tag{78}$$

where f is the observed frequency. For LIGO/Virgo in the 10–10^3 Hz band, the correction is $\delta\Psi \sim 10^{-3} \times \ln(100) \sim 5 \times 10^{-3}$, near the current sensitivity limit. LISA [33], with its 10^{-4}–10^{-1} Hz band and higher phase precision, could detect this signal in supermassive binary observations.

9.4 Large-scale CMB anisotropies

The active self-replication regime produces a modification of the CMB power spectrum at low multipoles ($\ell < 30$), where error bars are larger due to cosmic variance. The quantitative prediction is:

$$\left.\frac{\Delta C_\ell}{C_\ell}\right|_{\ell<30} \sim \frac{\Gamma_{\rm rep}}{H_0} \sim 10^{-3}, \tag{79}$$

in the right direction to partially explain the low-quadrupole anomaly observed by WMAP and Planck [3].

9.5 Correlation between black-hole density and local acceleration

The theory predicts a spatial correlation between the local black-hole density (estimated via gravitational lensing mapping) and the local cosmic acceleration. The signal, expressed as $\delta\rho_\Lambda/\rho_\Lambda \propto \rho_{\rm BH}/\rho_{\rm crit}$, is of order 10^{-5} on cluster scales ($\sim 10\,$Mpc), marginally detectable by Euclid [34] through cross-correlation statistics.

9.6 Comparative table of predictions

Table 3: Key observational predictions and experimental detectability.

Observable	**Magnitude**	**Experiment**	**Comments**
$w(z)$ deviation from -1	$\sim 10^{-3}$	DESI, Euclid	Detectable in 5 years
$H(z)$ deviation from ΛCDM	$\sim 10^{-3}$	DESI	Marginal at $z \sim 0.5$
GW phase	$\sim 5 \times 10^{-3}$	LISA	Detectable in SMBH binaries
C_ℓ CMB at low ℓ	$\sim 10^{-3}$	Planck (existing)	Consistent with quadrupole anomaly
$\rho_{\rm BH}$–ρ_Λ correlation	$\sim 10^{-5}$	Euclid	Marginal
Atomic clock drift	$\sim 10^{-69}\,{\rm s}^{-1}$	Far future	Below current sensitivity

10 Critical analysis and limitations

10.1 Potential objections and responses

10.1.1 Objection 1: Energy conservation

Objection: If ρ_ϕ grows exponentially, where does the energy come from? Does this not violate the first law of thermodynamics?

Response: In general relativity, *global* energy is not well defined in a non-stationary Universe; what is conserved is the energy-momentum tensor $\nabla^\mu T^{\rm tot}_{\mu\nu} = 0$ (Proposition 4.1). The energy apparently "created" by self-replication comes from the gravitational field: as ρ_ϕ grows, the *negative gravitational energy* of the cosmological horizon decreases by the same amount, conserving the total balance. This exchange is analogous to that occurring in the standard de Sitter universe, where the vacuum energy apparently "violates" Newtonian conservation but respects covariant conservation.

10.1.2 Objection 2: Compatibility with Planck

Objection: Planck 2018 bounds [3] impose $|w + 1| < 0.05$ at 95% confidence. Is the prediction $w = -1 + 10^{-3}$ compatible?

Response: Yes. The predicted deviation is 10^{-3}, an order of magnitude below the current bound. The theory is consistent with existing data and makes quantitative predictions that distinguish it from ΛCDM *and* from *phantom* models (where w is more negative than -1).

10.1.3 Objection 3: No-go theorems

Objection: The Maldacena–Nuñez no-go theorems [31] and others forbid self-acceleration in modified gravity theories with minimally coupled scalar fields. Do they apply here?

Response: The Maldacena–Nuñez theorems apply to theories derived from dimensional compactifications where the scalar is a modulus. The present theory *is not* a compactification theory;

the field ϕ is a fundamental field, not a modulus. Moreover, the non-minimal coupling $\xi = 1/6$ technically evades the hypotheses of the theorem.

10.1.4 Objection 4: Quantum stability of the potential

Objection: The $V_{\rm rep}(\phi)$ term with the step function Θ introduces a non-analyticity. Is the model quantum-mechanically stable?

Response: The non-analyticity is a technical device modeling a physical threshold: the activation of the tachyonic instability above ϕ_c. Microscopically, the step function is smoothed over a region of width $\Delta\phi \sim \phi_c/10$ without changing the physical predictions. One-loop radiative corrections preserve the qualitative structure of the potential, although they may renormalize ϕ_c by an $\mathcal{O}(1)$ factor.

10.1.5 Objection 5: Comparison with quintessence

Objection: What does this theory predict that quintessence does not?

Response: (i) Exponential growth of ρ_ϕ instead of decrease, (ii) asymptotic equation of state $w \to -1 + \epsilon$ with $\epsilon > 0$ (not $w < -1$), (iii) specific signature in gravitational waves via (78), (iv) $\rho_{\rm BH}$–ρ_Λ correlation. These predictions are falsifiable and distinguishable from conventional quintessence models.

10.2 Falsifiability

The theory is *falsifiable*: any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in the range $0 < z < 2$ refutes the model. Euclid and DESI, in their projected configurations for 2030, will reach this sensitivity. Additionally, a non-detection of the gravitational-wave phase (78) with LISA in supermassive black-hole binaries would refute the theory at 95% confidence.

10.3 Limitations of the model

We recognize three main limitations:

(i) **Microscopic origin of ϕ_c:** The threshold ϕ_c is a free parameter of the phenomenological model. A complete ultraviolet theory should derive ϕ_c from first principles, possibly from loop quantum gravity or string theory.

(ii) **Treatment of $\rho_{\rm BH}$:** The black-hole density is introduced as an external source. A more rigorous treatment should derive it dynamically from a hierarchical black-hole formation model coupled to the ϕ field.

(iii) **Complete numerical validation:** The analytical solutions presented here rely on approximations ($\Gamma_{\rm rep} \gg H$ regime, $\xi\phi^2 \ll M_{\rm Pl}^2$). A complete numerical validation of the system (50)–(49) without these approximations is left as future work.

10.4 Status with respect to the current consensus

The present theory sits at the boundary between standard cosmology and more speculative physics. It does not aim to replace ΛCDM — whose phenomenological success is formidable — but rather to propose an alternative theoretical framework that conceptually resolves the cosmic coincidence problem (dark energy grows, not decreases) and the cosmological constant problem (there is no vacuum contribution, there is dynamic production). Acceptance of the theory by the scientific community will ultimately depend on observational validation of its quantitative predictions, particularly the $w(z) \neq -1$ deviation with the specific functional form (76).

11 Conclusions

An alternative cosmological theory has been presented that identifies dark energy with a scalar field ϕ generated as a by-product of the gravitational combustion of matter, primarily at the event horizons of black holes. The field ϕ possesses a self-interaction potential $V_{\rm rep}(\phi)$ that activates above a critical threshold ϕ_c, inducing a local tachyonic instability that produces quantum self-replication at a rate $\Gamma_{\rm rep} = |m^2_{\rm eff}|/(4\pi)$. This mechanism, combined with FLRW cosmology, produces a super-accelerated expansion $a(t) \sim \exp[\exp(\Gamma_{\rm rep} t/2)]$ *without* violating the weak energy condition, distinguishing itself from conventional *phantom* models.

The theory is built on four explicit physical postulates (P1–P4) that translate into a complete scalar-tensorial Lagrangian (12), from which the modified Einstein–Klein–Gordon equations (20)–(29) are rigorously derived. Application to the FLRW Universe yields the modified Friedmann equations (Theorem 6.1) with a source term $\Gamma_{\rm rep}\rho_\phi$ in the continuity equation (55). The analytical solution in the dark-energy-dominated regime (Proposition 6.2) explicitly demonstrates the exponential growth of ρ_ϕ and $H(t)$, as well as the double-exponentiality of $a(t)$, without invoking a *phantom* equation of state $w < -1$.

The detailed microphysics (Section 8) characterizes the ϕ field through its quasiparticle spectrum (65), self-replication cross section (68), mean free time (69), effective temperature (71), number density (72), and coherence length (73). The model parameters (Table 2) are consistent with existing observational bounds and predict measurable deviations from ΛCDM in upcoming experiments (Table 3).

The central question motivating this theory — *why does dark energy self-replicate?* — is answered by the confluence of three physical facts: the gradual accumulation of the ϕ field by gravitational combustion, the tachyonic activation upon exceeding the threshold ϕ_c, and the spontaneous pair production by quantum instability. This triad constitutes a positive feedback mechanism analogous to a laser, where the background field pumps the quantum modes of the same field.

The theory is falsifiable. Any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in the range $0 < z < 2$ would refute it. Euclid, DESI, LISA, and CMB-S4, in their projected configurations for the coming decade, will reach the sensitivity needed to confirm or refute the quantitative predictions presented here.

The implications for future cosmology are profound. If the theory is confirmed, dark energy would cease to be a problem of the cosmological *constant* and would become a problem of the *dynamics* of quantum fields in curved spacetime, opening a direct observational window to the quantum physics of gravity. Quantum self-replication, in particular, could constitute the first detectable macroscopic phenomenon where quantum gravity plays an essential role.

The present theory is offered to the scientific community with epistemic humility: as a rigorous, mathematically self-consistent, falsifiable theoretical hypothesis motivated by a clear physical intuition. Its definitive validation or refutation will depend on observational progress in precision cosmology, in particular on measurements of $w(z)$ and on gravitational-wave signatures. The author is confident that the mathematical and conceptual structure developed here constitutes a useful contribution to the contemporary cosmological debate, regardless of the final verdict of the data.

A Notation and conventions

A.1 General conventions

- **Metric signature:** $(-,+,+,+)$, so that $g_{\mu\nu} = \mathrm{diag}(-1,+1,+1,+1)$ in the Minkowski limit.

- **Natural units:** $\hbar = c = k_B = 1$ unless otherwise indicated. The reduced Planck mass is $M_{\rm Pl} \equiv 1/\sqrt{8\pi G} \approx 2.435 \times 10^{18}\,\text{GeV}$.
- **Connection:** Levi-Civita, $\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\sigma\nu} - \partial_\sigma g_{\nu\rho})$.
- **Covariant derivative:** $\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\rho} V^\rho$.
- **Riemann tensor:** $R^\mu_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu_{\nu\sigma} - \partial_\sigma \Gamma^\mu_{\nu\rho} + \dots$ with Misner–Thorne–Wheeler sign convention.
- **Ricci scalar:** $R = g^{\mu\nu} R_{\mu\nu}$ with $R_{\mu\nu} = R^\rho_{\mu\rho\nu}$.
- **d'Alembert operator:** $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = (1/\sqrt{-g})\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi)$.

A.2 FLRW metric

The Friedmann–Lemaître–Robertson–Walker metric used is

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega^2\right],$$

with $k \in \{+1, 0, -1\}$. For $k = +1$ (closed Universe, used in this theory), $r \in [0, 1]$ and the total comoving volume is $2\pi^2$. The Ricci scalar is $R = 6(\dot{H} + 2H^2 + k/a^2)$.

A.3 Glossary of symbols

Symbol	Definition	Section
ϕ	Dark radiation scalar field	Sec. 2
m_ϕ	Mass of the ϕ field	Sec. 3
λ	Standard quartic self-interaction	Sec. 3
$\lambda_{\rm rep}$	Self-replication self-interaction	Sec. 3
ϕ_c	Tachyonic activation threshold	Sec. 3
ξ	Non-minimal curvature coupling ($= 1/6$)	Sec. 3
$\xi_{\rm BH}$	Black-hole–ϕ source coupling	Sec. 3
$\rho_{\rm BH}$	Black-hole mass-energy density	Sec. 3
$V_{\rm eff}(\phi)$	Total effective potential	Eq. (11)
$m^2_{\rm eff}$	Field-dependent effective mass	Eq. (33)
$\Gamma_{\rm rep}$	Quantum self-replication rate	Eq. (43)
ρ_ϕ	Energy density of the ϕ field	Eq. (54)
w_ϕ	Equation of state of the ϕ field	Sec. 6
$T^{(\phi)}_{\mu\nu}$	Energy-momentum tensor of the ϕ field	Eq. (21)

B Auxiliary derivations

B.1 B.1. Estimation of the non-BH source fraction

The matter density in high-curvature regions not associated with black holes (neutron stars, white dwarfs, dense gas) is of order $\rho_{\rm NS} \sim 10^6\,\text{g/cm}^3$ in the core of a neutron star. The typical curvature is $|R_{\rm NS}| \sim 10^{-14}\,\text{m}^{-2}$. The product $\rho_{\rm NS}\,|R_{\rm NS}|^{1/2}$ is $\sim 10^6 \times 10^{-7} = 10^{-1}\,\text{g cm}^{-3}\,\text{m}^{-1}$.

For a solar-mass black hole, $\rho_{\rm BH} \sim M_\odot/(4\pi r_s^3/3) \sim 10^6\,\text{g/cm}^3$ (near the horizon) and $|R_{\rm BH}| \sim 1/r_s^2 \sim 10^6\,\text{m}^{-2}$. The product is $\sim 10^6 \times 10^3 = 10^9\,\text{g cm}^{-3}\,\text{m}^{-1}$, four orders of magnitude larger.

Considering also that the total volume of supermassive black holes is comparable to the volume of their active accretion disks, the fraction of the source not associated with black holes is of order 10^{-4}, justifying Postulate P2.

B.2 B.2. Metric variation identities

The key identities used in Section 4 are proven:

Identity (14): From $\delta g = g\, g^{\mu\nu}\, \delta g_{\mu\nu} = -g\, g_{\mu\nu}\, \delta g^{\mu\nu}$, it follows that $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\, \delta g^{\mu\nu}$.

Identity (15): Using the Palatini formula, $\delta R_{\mu\nu} = \nabla_\rho \delta\Gamma^\rho_{\mu\nu} - \nabla_\nu \delta\Gamma^\rho_{\mu\rho}$, and the fact that $\delta\Gamma$ is a tensor, after integration by parts:

$$\delta\int \sqrt{-g}\, R\, \mathrm{d}^4x = \int \sqrt{-g}\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big)\, \delta g^{\mu\nu}\, \mathrm{d}^4x + \text{total term},$$

where the total term is discarded under the boundary conditions $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$.

Identity (17): Using $\delta(\sqrt{-g}\, R\phi^2) = \phi^2\, \delta(\sqrt{-g}\, R) + \sqrt{-g}\, R\, \delta(\phi^2)$, with $\delta(\phi^2) = 0$ (metric variation at fixed ϕ) and applying (15):

$$\delta\int \sqrt{-g}\, R\phi^2\, \mathrm{d}^4x = \int \sqrt{-g}\Big[\phi^2\Big(R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}\Big) + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2\Big]\, \delta g^{\mu\nu}\, \mathrm{d}^4x,$$

where the last term comes from integration by parts of the total term in (15).

B.3 B.3. Detailed computation of the energy-momentum tensor

$T^{(\phi)}_{\mu\nu}$ is defined via

$$\delta_g \mathcal{S}_\phi \equiv -\tfrac{1}{2}\int \sqrt{-g}\, T^{(\phi)}_{\mu\nu}\, \delta g^{\mu\nu}\, \mathrm{d}^4x.$$

Summing the contributions of the $\frac{1}{2}(\nabla\phi)^2$, $V_{\text{eff}}(\phi)$, $\xi R\phi^2/2$ terms and using (17):

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\, \nabla_\nu\phi - g_{\mu\nu}\Big[\tfrac{1}{2}(\nabla\phi)^2 - V_{\text{eff}}\Big] + \xi\Big[(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2 - \phi^2\, G_{\mu\nu}\Big]\,,$$

which is expression (21) of the main text.

B.4 B.4. FLRW components of the energy-momentum tensor

Substituting $\phi = \phi(t)$ in FLRW:

$$\begin{aligned} T^{(\phi)}_{00} &= \tfrac{1}{2}\dot\phi^2 + V_{\text{eff}} - 3\xi(H^2\phi^2 + 2H\dot\phi\phi + \ddot\phi\phi + \dot\phi^2), \\ T^{(\phi)}_{ij} &= a^2\Big[\tfrac{1}{2}\dot\phi^2 - V_{\text{eff}} + \xi(2H\dot\phi\phi + \ddot\phi\phi + \dot\phi^2 + (2\dot H + 3H^2)\phi^2)\Big]\, \delta_{ij}. \end{aligned}$$

Identifying $\rho_\phi = -T^0_0$ and $p_\phi = T^i_i/3$ yields the expressions used in the proof of Theorem 6.1.

B.5 B.5. Lyapunov analysis of the self-replication solution

The dynamical system in variables (ρ_ϕ, H) is:

$$\dot\rho_\phi = (\Gamma_{\text{rep}} - 3H(1+w_\phi))\rho_\phi, \qquad \dot H = \tfrac{1}{2}(\Gamma_{\text{rep}} - 3H(1+w_\phi))\, H,$$

where Friedmann's equation has been used to eliminate ρ_ϕ. The Jacobian at the solution of Proposition 6.2 has eigenvalues $\lambda_\pm = \Gamma_{\text{rep}}/2 \pm \sqrt{\Gamma^2_{\text{rep}}/4 + 4H^2(1+w_\phi)}$, with $\lambda_+ > 0$ (unstable) and $\lambda_- < 0$ (stable). The system is a hyperbolic saddle: generic trajectories converge asymptotically to the self-replication solution along the stable direction.

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Original contribution of this work

Authorship statement

The present work constitutes an original theoretical construction. The specific novel elements claimed as original contributions of this theory, developed in its entirety by the author, are the following:

(i) **Postulate of gravitational combustion (P1–P2):** the identification of dark energy as the inevitable by-product of matter falling into extreme gravitational fields, with black holes as the dominant sources. This analogy with chemical combustion (matter "burning" emitting "smoke") is, to the best of the author's knowledge, original to the present theory.

(ii) **Quantum self-replication mechanism (P4):** the introduction of an effective potential $V_{\text{rep}}(\phi)$ with threshold activation that induces a local tachyonic instability and spontaneous pair production of the same ϕ field. The rate $\Gamma_{\text{rep}} = |m_{\text{eff}}^2|/(4\pi)$ is rigorously derived from the Bogoliubov mode decomposition.

(iii) **Cosmological self-replication solution:** the analytical demonstration (Proposition 6.2) that the combination of self-replication with Friedmann equations produces double-exponential expansion without violating the weak energy condition, thus avoiding the *Big Rip* of *phantom* models.

(iv) **Falsifiable observational predictions:** the explicit equations for $w(z)$ (76), $H(z)$ (77), and the gravitational-wave phase (78), which quantitatively distinguish this theory from ΛCDM and from *phantom* models.

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The author expresses his willingness to maintain scientific correspondence on any aspect of this theory, as well as to collaborate with research groups interested in the development, refinement, or observational validation of the model. The theory is offered to the international scientific community as a serious, mathematically rigorous, and falsifiable hypothesis, in the spirit of the classical scientific method.

This work is dedicated to the public scientific domain in July 2026.